Probability and Random Processes

EES 315

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6.2 Independence

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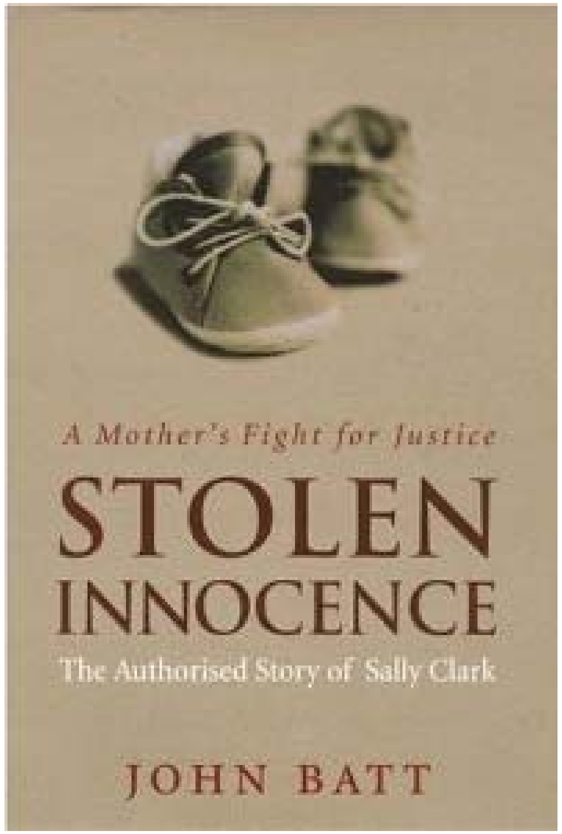
Murder and Accusation

Sally Clark

[http://www.sallyclark.org.uk/]
[http://en.wikipedia.org/wiki/Sally_Clark]
[http://www.timesonline.co.uk/tol/comment/obituaries/article1533755.ece]

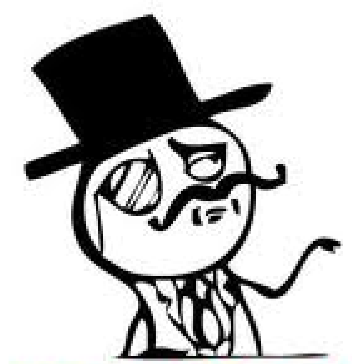
Sally Clark

- **Falsely accused** of the **murder of her two sons**.
  - Clark's first son died suddenly within a few weeks of his birth in 1996.
  - After her second son died in a similar manner, she was arrested in **1998** and tried for the murder of both sons.
- The case went to appeal, but the convictions and sentences were confirmed in 2000.
- Released in **2003** by Court of Appeal
- Wrongfully imprisoned for more than 3 years



Misuse of statistics in the courts

- Her prosecution was controversial due to **statistical evidence**
- This evidence was presented by a medical expert witness Professor Sir Roy Meadow,



$$\left(\frac{1}{8500}\right)^2 \approx 10^{-8}$$

- Meadow testified that the frequency of sudden infant death syndrome (SIDS, or "cot death") in families having some of the characteristics of the defendant's family is 1 in 8500.
- He went on to square this figure to obtain a value of 1 in 73 million for the frequency of two cases of SIDS in such a family.

Royal Statistical Society

- “This approach is, in general, **statistically invalid**.”
- “It would only be valid if SIDS cases arose **independent**ly within families, an assumption that would need to be justified empirically. “
- “There may well be unknown genetic or environmental factors that predispose families to SIDS, so that **a second case within the family becomes much more likely**.”

[http://www.rss.org.uk]

Engineering Ethics: IEEE Code of Ethics

We, the members of the IEEE, in recognition of the importance of our technologies in affecting the quality of life throughout the world, and in accepting a personal obligation to our profession, its members and the communities we serve, do hereby commit ourselves to the highest ethical and professional conduct and agree:

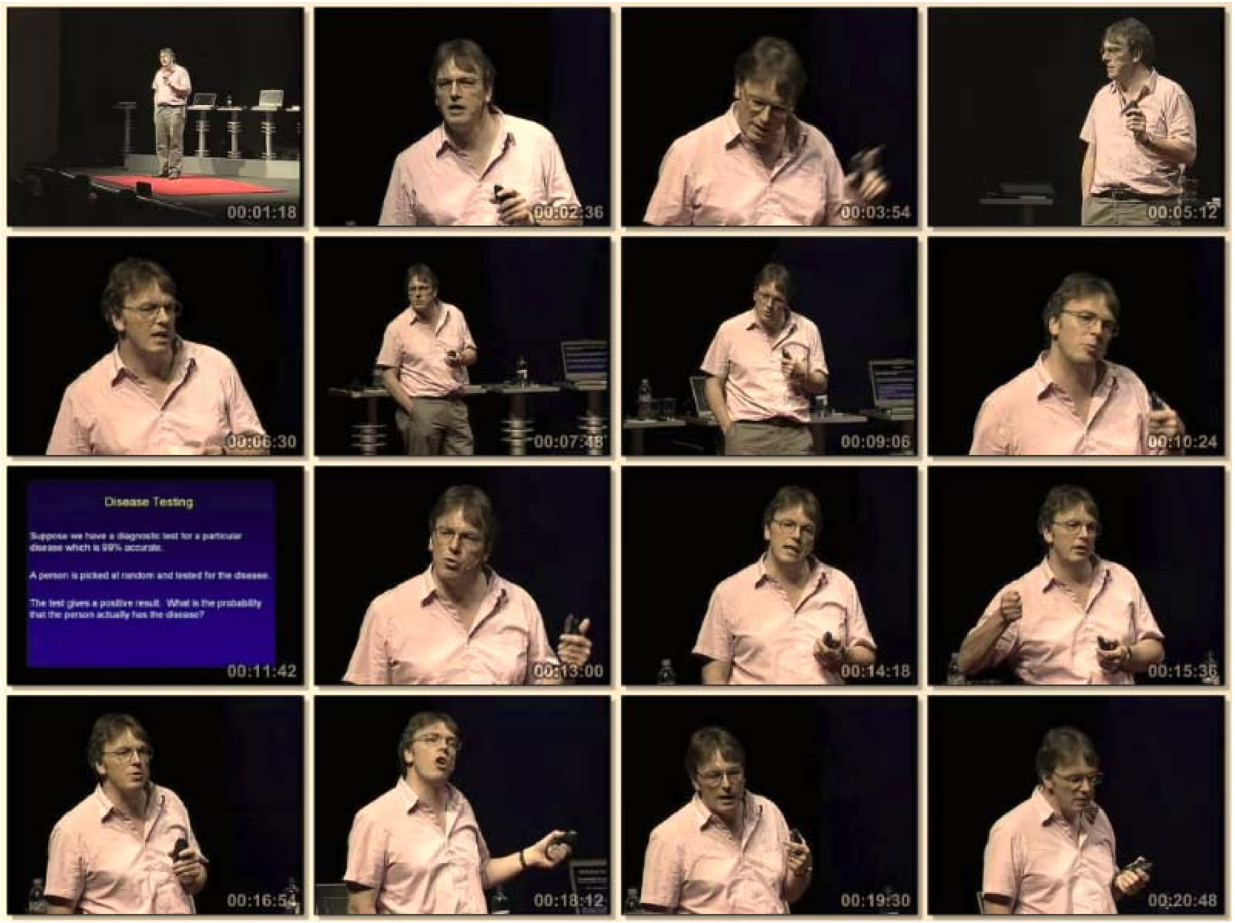
1. to accept responsibility in making decisions consistent with the safety, health, and welfare of the public, and to disclose promptly factors that might endanger the public or the environment;
2. to avoid real or perceived conflicts of interest whenever possible, and to disclose them to affected parties when they do exist;
3. to be honest and realistic in stating claims or estimates based on available data;
4. to reject bribery in all its forms;
5. to improve the understanding of technology; its appropriate application, and potential consequences;
6. to maintain and improve our technical competence and to **undertake technological tasks for others only if qualified by training or experience**, or after full disclosure of pertinent limitations;
7. to seek, accept, and offer honest criticism of technical work, to acknowledge and correct errors, and to credit properly the contributions of others;
8. to treat fairly all persons regardless of such factors as race, religion, gender, disability, age, or national origin;
9. to avoid injuring others, their property, reputation, or employment by false or malicious action;
10. to assist colleagues and co-workers in their professional development and to support them in following this code of ethics.

Epilogue

- Clark's release in January 2003 prompted the Attorney General to order a review of hundreds of other cases.
- **Two other** women convicted of murdering their children had their convictions overturned and were released from prison.
- Trupti Patel, who was also accused of murdering her three children, was acquitted in June 2003.
- In each case, Roy Meadow had testified about the unlikelihood of multiple cot deaths in a single family.

How Juries Are Fooled by Statistics

- By Peter Donnelly



Professor of Statistical Science (Dept Statistics) at University of Oxford

@ 11:15-13:50 Disease Testing
@ 13:50-18:30 Sally Clark

http://www.youtube.com/watch?v=kLmzxmRcUTo
http://www.stats.ox.ac.uk/people/academic_staff/peter_donnelly

Prosecutor’s Fallacy

- Aside from its invalidity, figures such as the 1 in 73 million are very easily misinterpreted.
- Some press reports at the time stated that this was the chance that the deaths of Sally Clark's two children were accidental.
- This (mis-)interpretation is a serious error of logic known as the **Prosecutor's Fallacy**.
- The jury needs to weigh up two competing explanations for the babies' deaths: 1) SIDS or 2) murder.
- Two deaths by SIDS or two murders are each quite unlikely, but one has apparently happened in this case.
- What matters is the relative likelihood of the deaths under each explanation, not just how unlikely they are under one explanation (in this case SIDS, according to the evidence as presented).

Review: Independence (two events)

(If one of them is satisfied, all of them are satisfied.)

- The following statements are equivalent

1) Events A and B are **independent**
2) $A \perp\!\!\!\perp B$
3) $A \perp\!\!\!\perp B^c$
4) $A^c \perp\!\!\!\perp B$
5) $A^c \perp\!\!\!\perp B^c$
6) $P(A \cap B) = P(A)P(B)$
7) $P(A \cap B^c) = P(A)P(B^c)$
8) $P(A^c \cap B) = P(A^c)P(B)$
9) $P(A^c \cap B^c) = P(A^c)P(B^c)$

Formal mathematical definition for independence

- Furthermore, if $P(A) > 0$ $(\neq 0)$ and $P(B) > 0$ $(\neq 0)$, then the above statements are also equivalent to

10) $P(A|B) = P(A)$
11) $P(B|A) = P(B)$

Practical definitions for independence

Review: Independence (> 2 events)

- **Three events**: Need to check via 2^3-3-1 = 4 conditions:

Independence for three events needs four conditions.

$$\begin{cases} P(A \cap B) = P(A)P(B) \\ P(A \cap C) = P(A)P(C) \\ P(B \cap C) = P(B)P(C) \\ P(A \cap B \cap C) = P(A)P(B)P(C) \end{cases}$$

Pairwise independence for three events is defined by the first three conditions

- ***n*** **events**: Events $A_1, A_2, A_3, \dots, A_n$ are independent if and only if for all distinct indices $i_1, i_2, i_3, \dots, i_k$, $\mathbf{2 \leq k \leq n}$

$$P(A_{i_i} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_i})P(A_{i_2}) \cdots P(A_{i_k}).$$

  - Need to check $2^n - n - 1$ conditions